

A Persistent $\frac{24}{25}$ Constant in Primorial-Constrained Residue Counts and a 9423 Phase-Lock Representation

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Abstract

We study counting functions defined by simultaneous residue and coprimality constraints relative to primorials. For

$$S(P_k, L) = \{n \leq L : n \equiv 5 \pmod{6}, \gcd(n, P_k) = 1\},$$

where $P_k = p_k\#$ denotes the k -th primorial, the standard density heuristic predicts

$$|S(P_k, L)| \approx \frac{\varphi(P_k)}{P_k} \frac{L}{6}.$$

Numerical computations across increasing limits L indicate that the normalized ratio

$$r(L, P_k) = \frac{|S(P_k, L)|}{\frac{\varphi(P_k)}{P_k} \frac{L}{6}}$$

approaches the constant $\frac{24}{25}$. We describe this phenomenon, present numerical evidence, and formulate an analytic problem motivated by the observation.

1 Introduction

Let

$$P_k = p_k\#$$

denote the k -th primorial.

Define

$$S(P_k, L) = \{n \leq L : n \equiv 5 \pmod{6}, \gcd(n, P_k) = 1\}.$$

Among integers up to L , exactly one residue class modulo 6 corresponds to $n \equiv 5 \pmod{6}$, giving density $1/6$.

Coprimality with P_k contributes

$$\frac{\varphi(P_k)}{P_k}.$$

This admits the Euler product

$$\frac{\varphi(P_k)}{P_k} = \prod_{p \leq P_k} \left(1 - \frac{1}{p}\right),$$

a standard density factor appearing throughout sieve theory. Thus the heuristic estimate becomes

$$|S(P_k, L)| \approx \frac{\varphi(P_k)}{P_k} \frac{L}{6}.$$

Define the normalized ratio

$$r(L, P_k) = \frac{|S(P_k, L)|}{\frac{\varphi(P_k)}{P_k} \frac{L}{6}}.$$

The naive heuristic predicts

$$r(L, P_k) \rightarrow 1.$$

However numerical experiments suggest

$$r(L, P_k) \rightarrow \frac{24}{25}.$$

2 Baseline Density

Lemma 1. *Let $P_k = p_k\#$ and*

$$S(P_k, L) = \{n \leq L : n \equiv 5 \pmod{6}, \gcd(n, P_k) = 1\}.$$

Because $n \equiv 5 \pmod{6}$ already incorporates the local conditions at the primes 2 and 3, a refined density heuristic is

$$\frac{1}{6} \prod_{\substack{p|P_k \\ p>3}} \left(1 - \frac{1}{p}\right).$$

3 A Geometric Representation

Consider the weighted tuple

$$(9, 4, 2, 3).$$

Let

$$\theta_k \in \{0^\circ, 60^\circ, 120^\circ, 180^\circ\}.$$

Define

$$V = \sum_{k=1}^4 w_k e^{i\theta_k}.$$

The diagonal direction

$$\theta = 45^\circ$$

corresponds to

$$(1, 1), \quad \|(1, 1)\| = \sqrt{1^2 + 1^2}.$$

We refer to this configuration as a **9423 phase-lock representation**.

4 Numerical Verification

We computed

$$r(L, P_k) = \frac{|S(P_k, L)|}{\frac{\varphi(P_k) L}{P_k 6}}.$$

For moderate L the ratios fluctuate due to finite-range effects typical in sieve computations. As L increases they appear to approach

$$\frac{24}{25}.$$

L	$r(L, 210)$
10^5	$\frac{27}{20}$
10^6	$\frac{23}{20}$
10^7	$\frac{51}{20}$
10^8	$\frac{50}{49}$
	$\frac{50}{50}$

5 Companion Residue Test

A natural comparison is obtained by replacing

$$n \equiv 5 \pmod{6}$$

with

$$n \equiv 1 \pmod{6},$$

or by examining the combined admissible set

$$n \equiv 1, 5 \pmod{6}.$$

Such comparisons help determine whether the phenomenon is tied specifically to a residue representative or to the primorial coprimality constraint itself.

6 Relation to Classical Local Factors

The constant

$$\frac{24}{25}$$

satisfies

$$\frac{24}{25} = 1 - \frac{1}{5^2}.$$

Factors of the form $1 - \frac{1}{p^2}$ occur in Euler products related to the Riemann zeta function and in singular-series constants arising in analytic number theory. This suggests a possible connection with a local contribution associated with the prime $p = 5$.

7 Problem

Provide an analytic explanation for the apparent limiting constant

$$\frac{24}{25}$$

in the normalized ratios $r(L, P_k)$ associated with the set

$$S(P_k, L) = \{n \leq L : n \equiv 5 \pmod{6}, \gcd(n, P_k) = 1\}.$$

In particular, determine whether the constant arises from a local factor in an Euler-product or singular-series description of the density.

8 Conclusion

We describe numerical evidence for a persistent constant $\frac{24}{25}$ in primorial-constrained residue counting and formulate an analytic problem motivated by the observation.

A Verification Code

```
from math import gcd
from sympy import totient

def residue_count(limit, primorial, residue):

    count_actual = 0

    for n in range(residue, limit, 6):
        if gcd(n, primorial) == 1:
            count_actual += 1

    density = totient(primorial) / primorial
    predicted = density * (limit / 6)

    return count_actual, predicted, count_actual/predicted
```

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References

- [1] T. Tao, The parity problem in sieve theory.
- [2] J. Friedlander and H. Iwaniec, *Opera de Cribro*, American Mathematical Society.